# EFFECTS OF INSTRUCTION ON STRATEGY TYPES CHOSEN BY GERMAN 3RD-GRADERS FOR MULTI-DIGIT ADDITION AND SUBTRACTION TASKS: AN EXPERIMENTAL STUDY

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*In an experimental study, we implemented two instructional approaches to teach 73 3rd graders from 17 school classes adaptive strategy use. The explicit approach encompassed the explicit teaching and practicing of selected strategies, whereas the problem-solving approach emphasised the analysis of task characteristics and the individual generation of strategies. Results from post- and follow-up tests after the intensive one-week intervention did not yield significant differences between the two approaches in the efficency and in the accuracy of the applied strategies. In this contribution we report an additional analysis of the data examining the types of strategies the students chose. Although both groups used efficient strategies, it turned out that they differed significantly in the types of strategies they chose.*

## INTRODUCTION

Adaptive strategy use in arithmetic, i.e., solving computation tasks efficiently by flexibly choosing an “advantageous” strategy, is considered as an important aspect of mathematics education. Although the standard (written) algorithms for the basic arithmetic operations still play a prominent role in arithmetic education, in many countries text books and primary school curricula also address students’ competence to adequately use different strategies for solving arithmetic tasks. However, as empirical studies repeatedly revealed, the acquisition of such an adaptive expertise is quite challenging and empirical findings indicate unsatisfactory results for primary school students (e.g. Heinze, Marschick, & Lipowsky, 2009; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009). Accordingly, specific instructional approaches are discussed to organise effective learning opportunities to support students. These approaches are based on different learning theories and follow different assumptions about the acquisition of adaptive expertise. However, there are hardly empirical studies on the comparison of these instructional approaches for students’ adaptive strategy use.

## THEORETICAL BACKGROUND AND EMPIRICAL FINDINGS

### Strategy types and adaptive strategy use

For an empirical examination of students’ strategies it is necessary to choose a category framework to make the observed strategies accessible for a deeper analysis. Arithmetic computation strategies for multi-digit addition and subtraction can be categorised in various ways (see an overview in Threlfall, 2002, pp. 33ff.). In prominent German mathematics education books the categorisation in Table 1 is described. It distinguishes five main types of strategies for addition and subtraction problems, each type covers several strategies. For example, the jump strategy type encompasses jump strategies

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which successively add the hundreds, tens and units of the second summand or the other way round the units, tens and hundreds of the second summand or the two strategies which analogously decompose the first summand. The types jump and split strategy encompass universal strategies which can be applied for all addition and subtraction problems. [It is an open discussion how to deal with the split strategies in case of subtraction problems with regrouping. Some of the German textbooks introduce a split strategy but avoid the notation of intermediate (negative) results.] The strategies of the other three types are advantageous only for specific problems and cannot be applied efficiently in general. All these strategy types are idealised strategy types in the sense that children obviously are quite creative and generate strategies of further types, especially by combining two or more strategies of different types (e.g., Selter, 2001).

| Jump strategy | Split strategy | Compensation strategy | Simplifying strategy | Indirect addition* |
|---|---|---|---|---|
| 123 + 456 = *579* | 123 + 456 = *579* | 527 + 398 = *925* | 527 + 398 = *925* | 701 – 698 = *3* |
| 123 + 400 = 523 | 100 + 400 = 500 | 527 + 400 = 927 | 525 + 400 = 925 | 698 + 3 = 701 |
| 523 + 50 = 573 | 20 + 50 = 70 | 927 – 2 = 925 | | |
| 573 + 6 = 579 | 3 + 6 = 9 | | | |

Table 1: Idealised types of computation strategies with examples.
[*The indirect addition strategy is for subtraction problems only.]

As in our previous research, we describe students' competence for an adaptive strategy use by the efficiency of the applied strategy for a given task (Grüßing, Schwabe, Heinze, & Lipowsky, 2013). Here, we take into account two perspectives: For a student solving a given arithmetic task, one can check (1) from a mathematical perspective which strategy (or strategies) need(s) the smallest number of solution steps and (2) from a psychological perspective how much cognitive effort different solution steps require, which obviously depends on the knowledge and skills the individual has acquired so far (probably biased by affective variables like self-efficacy). Based on these criteria, we can define normatively which strategies are considered as efficient for a student solving a given arithmetic task and which are not. This norm is not restricted only to the properties of a given task but as in other studies like Klein, Beishuizen, and Treffers (1998) takes into account knowledge and skills of the considered student. Accordingly, in our research with 3rd graders, we first identify the range of strategies which can be expected by the group of students under investigation (i.e., strategy repertoire in the sense of declarative knowledge as well as the fluent and accurate application of these strategies with low cognitive effort in the sense of procedural knowledge). Then for these strategies we analyze how they fit to the characteristics of a given task and, thus, provide a short solution. However, it has to be mentioned that there might be other influental factors beyond these criteria. For example, Verschaffel, Luwel, Torbeyns, and Van Dooren (2009) suggest the context (in the sense of socio-mathematical norms) as possible factor when a teacher in her/his class implicitly conveys a reference framework which favors specific strategies. This problem is addressed in our sampling procedure by selecting only a few students from each class and, thus, reducing the influence of shared socio-mathematical norms.

**Teaching adaptive strategy use**

Empirical findings repeatedly revealed a low proficiency of primary school students in adaptive strategy use. In particular, many students have one or two favorite strategies – mostly one for addition and a different one for subtraction (in Germany: jump strategy for subtraction, split strategy for addition, Heinze et al., 2009). Moreover, most students solely use the standard algorithms after they have been introduced (e.g., Selter, 2001). Based on these results the question arises how to teach the adaptive strategy use to students.

In the literature, we find the traditional approach and so-called reform-based approaches (e.g., Verschaffel et al., 2009). In the traditional approach firstly only one strategy – in general, the jump strategy – is taught to and practiced by the students so that it can be applied accurately as a routine procedure. After that sometimes other strategies are mentioned in a sense that there exist helpful „computation tricks" for specific tasks. The reform-based approaches can be divided in two quite different types which we denote as explicit approach and problem-solving approach (see Heinze et al., 2009 for details). In the *explicit approach* firstly students invent their own strategies in an introductory phase. After that the teacher structures and reduces the diversity of invented strategies to a set of main strategies (cf. Table 1) which are successively practiced by the students. Finally, the adaptive strategy use is emphasised through solving tasks and discussing different solutions. An example for this explicit approach is the realistic program design as implemented in the study by Klein et al. (1998).

In contrast to the explicit approach, the *problem-solving approach* does not follow the idea of selecting a strategy from an individual strategy repertoire (cf. Threlfall, 2002). There are no official strategies introduced or named by the teachers. Students consider each arithmetic task as a new problem and generate a specific solution strategy for this problem (based on their knowledge and experience and on the task characteristics). Hence, students get many opportunities to analyze task characteristics, to solve problems and to discuss the efficiency of the students' solution strategies. Accordingly, they can accumulate knowledge on task characteristics and on skills in applying and judging individual strategies so that they will optimise their adaptive strategy use step by step.

Currently, we do not have much empirical evidence for the effectiveness of these instructional approaches. The one-year quasi-experimental study of Klein et al. (1998) indicates an advantage of the explicit approach in comparison to the traditional approach. Heinze et al. (2009) report that 3rd-graders taught by textbooks following the explicit or the problem-solving approach outperform 3rd-graders taught by textbooks following the traditional approach. Moreover, this study and also the findings of Torbeyns, De Smedt, Ghesquière, and Verschaffel (2009) indicate that high achieving students can also reach a high level of adaptive expertise when they are taught by the traditional approach. Grüßing et al. (2013) report a controlled experimental study comparing idealised implementations of the explicit and the problem-solving approach (see 3.1 for the design). Their results suggest that there are no significant differences in the short term and long term effects of both reform-oriented approaches on the competence of adaptive and accurate strategy use.

## RESEARCH QUESTION AND METHODOLOGY

Although there exists only a small number of empirical studies on instructional approaches teaching the adaptive strategy use, it seems that reform-oriented approaches are more beneficial than the traditional approach. Interestingly, our results in Grüßing et al. (2013) indicate no difference in the effectiveness of the explicit and the problem-solving approach. Since the approaches have quite different theoretical assumptions about the acquisition of adaptive expertise and since they strongly differ in the derived teaching activities in the mathematics classroom, we conducted a further fine grained analysis of the data to answer the following research questions:

Do the children of both groups

- differ in their choice of specific strategies after the intervention (i.e. in the posttest and the follow-up tests)?
- develop differently during and after the intervention?

### Sample, design and instruments

This section presents the main information of the experimental study as it was already described in Grüßing et al. (2013). The sample of the study comprised 79 randomly chosen 3$^{rd}$-graders (9-10 years old) from 17 classes of German primary schools from which we included 73 in this additional analysis. Six students were excluded because already in the pretest they used almost exclusively the efficient compensation strategy or the dominant written algorithms (i.e., their pretest results showed that they were more than 6 months ahead of the grade 3 curriculum, possibly due to out of school support). In a first step, the 73 children were randomly allocated to one of the two instructional approaches and after that the groups were parallelised according to general cognitive abilities, general mathematics achievement and socio-economic status.

The intervention was organised as a one-week course at our research institute during fall holidays. The overall intervention time was equivalent to 16 schools lessons (45 min) and accompanied by breaks for playing games and lunch. The lessons were taught by two trained research assistants following detailed teaching scripts of the explicit and the problem-solving approach (a short overview is given in Table 2). Expert ratings confirmed that teaching scripts and material mirrored the two approaches and that the comparison is fair. To limit the group size, we had two student groups for each approach (one group was taught in the first and one in the second holiday week). To control for teacher effects, both teachers taught each approach once.

| Day | Explicit approach | Problem-solving approach |
|---|---|---|
| 1 | Repetition of numbers up to 1000 and introduction of small group discussions | |
| 2 | Discovery & practice of jump and split strategy, small group discussions of individual solutions | Distance of given numbers, decomposing numbers, categorising tasks in easy, smart[1] and other tasks |
| 3 | Discovery & practice of indirect addition, compensation & simplifying | Categorising tasks, generation of easy and smart tasks |
| | Solving tasks and comparing solutions in small group discussions | |
| 4 | Repetition of all strategies | Categorising tasks and discussing individual criteria for categorisation |
| | Solving tasks and comparing solutions in small group discussions | |
| 5 | Post-tests and interviews[2], closing session | |

1. "Easy tasks" can be solved immediately (e.g., 150 + 230), "smart tasks" easily by a specific strategy (e.g., 329 + 141). Obviously, the allocation of tasks depends on the individual.
2. We also conducted interviews which are not discussed in this paper.

Table 2: Content of the one-week holiday course for both approaches

Data for adaptive strategy use was collected by trained university assistants with a pre-test 2 weeks before the intervention (T1), an immediate post-test (T2) and two follow-up tests after 3 (T3) and 8 months (T4). The test T4 was administered after the students learned standard algorithms for addition and subtraction. Each test consisted of 8 multi-digit addition and subtraction tasks suggesting specific strategies as efficient solutions (e.g. compensation, simplifying, etc., see Table 1). The tests were linked by anchor items: consecutive tests had 6 common items and 4 anchor items were used in all tests (403-396, 1000-991, 398+441, 502+399).

The item solutions were categorised by the strategies the students used for their solution. We started with a fine-grained system of 21 strategy categories which we retrieved from the literature and from theoretical analysis supplemented by some "bottom-up" strategy categories which frequently occurred in the student solutions. The category system included the main strategy types from Table 1 (e.g. jump strategy, split strategy, compensation strategy etc.) with several subcategories (e.g. jump strategy starting with units). For each test, the allocation of student solutions to categories was conducted independently by two trained research assistants (all Cohen's $\kappa > .70$) followed by a consensual agreement in case of different ratings.

For answering the research questions, we applied Chi-squared tests for homogeneity. This statistical test allows determining whether the distribution of the chosen strategies is significantly related to the group variable, i.e. whether the distribution of observed strategies in the explicit group and in the problem-solving group are similar or not. Due to mathematical prerequisites for the statistical Chi-squared tests, we had to merge categories in a senseful way to avoid too many small cell frequencies. We finally used 11 strategy categories for the first and 7 categories for the second research question.

## RESULTS

### Effects of the teaching approaches on the chosen strategies

To analyse the effects of the one-week intervention, we compare the chosen strategies of the children in the explicit group with those of the children in the problem-solving group separatley for the pretest (T1), the posttest (T2) and the follow-up tests (T3, T4). The categories and the frequencies of the chosen strategies in each group in each test are presented in Table 3. In the pretest the two groups did not differ significantly whereas in all other tests we found significant differences with moderate effect sizes. Concerning the specific efficient strategies for the test items, we can observe that immediately after the intervention the explicit group preferred strategies of the types indirect addition and simplifying whereas the problem-solving group preferred strategies of the type compensation. In the follow-up tests after three and eight months, the preference for the indirect addition and simplifying strategies in the explicit group is lost whereas the problem-solving group still keeps stable in the preference of the compensation type strategies.

### Change of preferred strategies over time

For the second research question, we analyzed the development of the strategy distribution in both groups separately. Due to space limitations we cannot present the table in this contribution. The analysis is based on the four anchor items so that we can compare the three time intervals. For both groups we found significant differences between two consecutive tests except the interval T2-T3 in the problem-solving group. The associated effect sizes indicated that – as expected – in both groups striking changes occurred during the intervention phase T1–T2 (Cramér's $V$ is .54 for the explicit and .46 for the problem-solving group) and in the phase T3-T4 when the dominant standard algorithms are taught in the regular mathematics classroom (Cramér's $V$ is .45 for the explicit and .46 for the problem-solving group). Remarkable is that during the three months after the intervention (T2–T3), the students of the problem-solving approach remained comparatively stable in their strategy choice whereas in the explicit group the use of specific strategies trained in the intervention (indirect addition, compensation, simplifying) decreased.

| Frequencies | T1 (pretest) | T2 (posttest) | T3 (after 3 months) | T4 (after 8 months) |
|---|---|---|---|---|

| | Explicit | Problem solving | Explicit | Problem solving | Explicit | Problem solving | Explicit | Problem solving |
|---|---|---|---|---|---|---|---|---|
| Written algorithms | 5 | 4 | 3 | 11 | 20 | 10 | 118 | 103 |
| Split strategy | 32 | 21 | 55 | 9 | 59 | 14 | 31 | 9 |
| Short split | 7 | 7 | 1 | 1 | 1 | 4 | 6 | 7 |
| Jump strategy | 101 | 109 | 42 | 42 | 48 | 38 | 4 | 11 |
| Short jump | 43 | 51 | 23 | 49 | 30 | 69 | 9 | 29 |
| Combination split & jump | 40 | 42 | 12 | 39 | 13 | 11 | 19 | 6 |
| Indirect addition | 5 | 4 | 61 | 26 | 20 | 24 | 9 | 11 |
| Compensation[1] | 5 | 6 | 29 | 55 | 35 | 65 | 23 | 75 |
| Simplifying[1] | | | 45 | 18 | 16 | 12 | 21 | 21 |
| Purely mental | 11 | 19 | 14 | 17 | 30 | 10 | 19 | 11 |
| Not assignable | 26 | 8 | 7 | 3 | 7 | 3 | 3 | 1 |
| Total[2] | 275 | 271 | 292 | 270 | 279 | 260 | 262 | 275 |
| $\chi^2$ | $\chi^2(9, N = 546) = 15.27$ | | $\chi^2(10, N = 562) = 96.19$ | | $\chi^2(10, N = 539) = 70.52$ | | $\chi^2(10, N = 537) = 58.04$ | |
| *p* | .084 | | < .001 | | < .001 | | < .001 | |
| Cramér's *V*[3] | .17 | | .41 | | .36 | | .33 | |

[1] For T1 compensation and simplifying were merged to avoid too many low cell frequencies

[2] Sample N = 584 (73 students times 8 items for each test) was reduced by missings (single items not processed or single students did not participate in one test); the subsample of 63 students which participated in all four tests yields similar results.

[3] Effect size Cramér's *V*: < .3 weak relation, .3-.5 moderate relation, >.5 strong relation between the variables

Table 3: Comparisons of the strategy distributions of the two groups at T1-T4

## DISCUSSION

The results give further insight into the relation between instructional characteristics and the strategy choice of students. As mentioned, the two instructional approaches which follow different educational philosophies have similar positive effects on students' competence to find efficient solutions for given arithmetics tasks (Grüßing et al., 2013). However, the effects of the instructional approaches are quite different if we take a qualitative perspective. After the intervention (T2), students of the explicit group use more frequently the demanding specific strategies (categories "simplifying" and "indirect addition") which were explicitly taught. However, the frequency of these strategies decreases in the following three months, perhaps, because they were learned only superficially. In contrast, students of the problem-solving group use more frequently and stable self-invented strategies after the intervention (short jump, compensation, combined strategies). As expected (cf. Selter, 2001), at T4 the dominant written algorithms are the main strategy type for both groups (45% in explicit, 37% in the problem-solving group). Nevertheless, eight months after the intervention students of the problem-solving group still choose frequently (self-invented) compensation strategies.

Summarising the findings, it seems that, firstly, the availability of an individually acquired strategy repertoire is more sustainable if the strategies are self-invented by the students. Secondly, it turns out that important strategies like indirect addition or simplifying are quite demanding and many children cannot invent such strategies on their own so that an adequate support is needed.

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